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OPTIMAL PREDICTION PERIODS FOR NEW AND OLD VOLATILITY INDEXES IN USA AND GERMAN MARKETS

Abstract: In 1993, the Chicago Board of Options Exchange (CBOE) introduced the VXO, a volatility index based on implied volatilities on S&P 100 index. In 2003, the CBOE changed their volatility index design and introduced the VIX in order to enhance its economic significance and to facilitate hedging. In this paper, using data from the USA and the German stock markets, we compare the forecasting capability of the volatility indexes with that of historical volatility and conditional volatility models.

Following this analysis, we have studied whether it may be the case that volatility indexes forecast the realized volatilities more accurately for a different period to 30 (or 45) days, attempting to answer the question: what time horizon is the informational content of volatility indexes best adjusted for? The optimal prediction period of each volatility index (VXO, VIX, VDAX and V1X) in terms of coefficient of determination is analysed.

The results identify a difference between the observed optimal forecasting period and the theoretical one. This could be explained from different perspectives such as the index’s design, investor cognitive bias or overreaction.

Keywords: VIX, VDAX, forecasting, realized volatility, maturity.

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1. INTRODUCTION

Volatility is a key concern in financial markets; a lot of effort has been put into predicting the future volatility of asset prices to try to take advantage of market opportunities. Mainly, practitioners have used two predictors: the historical volatility (HV), which is a statistical measure of past price movements of the assets; and the implied volatility (IV), which is calculated from prices of options traded in the market, considering a given valuation model and assuming that the rest of parameters involved are known. Therefore, HV is a measure of how prices have moved in the past and its usefulness is based on considering that such volatility might be repeated again in the future. By contrast, IV is a measure based on traded options' prices, which includes information on investors' sentiment about the underlying volatility until the maturity of the option. Most studies indicate that IV is the best indicator of future volatility, for example Blair et al. (2001) find that the most relevant information is found in the IV.

However, the use of IV also involves several problems, there are call and put options, there are different strike prices and at the money (ATM) options are not always available. Furthermore, maturity decreases as time progresses (if volatility of today is referred to a 25-day period, the volatility of tomorrow will be for a 24 day period and so on), whereas the reference period of the IV is assumed constant. Thus, using IV as a predictor of future volatility is not a straightforward task. In this sense, in 1973 the possibility of creating a volatility index arose, which could bring together information from a set of options and could homogenize the prediction period. Different ways of calculating this index has been proposed. In 1993, the Chicago Board of Exchange (CBOE) developed the VIX as a measure of the level of implied volatility of the USA market. Subsequently, other markets have joined this initiative and set up their own volatility indexes, among these are the German, Russian and Japanese ones.

The original volatility index design proposed by CBOE suffered from some drawbacks, mainly the difficulties to hedge the corresponding volatility swap contract, so in 2003, CBOE changed the methodology for calculating the volatility index and the rest of the markets decided to adopt their design. Thus, a first aim of this paper is to analyse how the change in methodology has increased or decreased the predictive power of the index. We focus on volatility indexes of the American and German markets, both in its original version (VXO and VDAX respectively) and the ones currently used (VIX and V1X, respectively), of which large historical series are available, allowing comparisons to be made to determine whether the change in the index calculation methodology has led to an improvement or not in its ability to predict realised volatility.

In addition, Car and Wu (2006) and Luo and Zhang (2012), among others, have studied the information contained in the volatility index about realized volatility, finding that the index provides more information than the historical volatility or the volatilities obtained with a conditional volatility model . However, in all these studies, a relationship is established between the index and realized volatility for the same time period (30 calendar days). Luo and Zhang (2012) go further and consider different time periods for both the VIX and realized volatility, though always establishing a relationship of equal maturities between both values. In this paper, we propose to change the period of calculation for realized volatility trying to find the period that best corresponds to the volatility indexes' forecasting. This could be quite different from the expected ones, since the value of the volatility index is a weighting of volatilities (or variances) related to options whose maturities do not match the standard period for which the index is designed.

The rest of the paper is structured as follows. In the next section, the four analysed indexes (VXO, VIX, VDAX and V1X) are briefly introduced, their historical behaviour is discussed and a descriptive analysis of the series is performed. The third section compares the predictive ability of volatility indexes with the historical volatility model and the conditional volatility GARCH(1,1) model. Section four presents the main contribution of the paper: the determination of the optimal prediction period for the volatility indexes in terms of coefficient of determination. Finally, in the conclusion, the most relevant results of the analysis are highlighted.

2. VOLALITY INDEXES IN FINANCIAL MARKETS

The first market to introduce a volatility index was the CBOE that adopted the VIX in February 1993 as an indicator of the volatility of options on the S&P 100 (OEX). It is mainly based on the proposal of Cox and Rubinstein (1985) in relation to the weight of maturities and strike prices. Later, in September 2003, the new VIX was introduced, based on the S&P 500 (SPX) and using the price of the options instead of IV to build the index. The old VIX was renamed as VXO, preserving the original name VIX for the new one.

Gradually, other countries have incorporated volatility indexes into their markets. Thus, in the German market, the VDAX is calculated from 1994 in a fairly similar form to the old VIX, and since 2007 it has been calculated according to the new methodology of the new VIX. Since October 1997, MONEP has calculated two indexes of volatility on the CAC-40, the VX1 and VX6; building on the work of Brenner and Galai (1989), and since 2007 has also used the new VIX methodology for calculating the VCAC. More recently, other countries such as Canada (TSX VIX), Japan (VXJ), Russia (RTSVX), Australia (ASX VIX) or India (India VIX) have incorporated volatility indexes into their markets, following the same methodology of the CBOE for VIX in its new version.

2.1 The design of volatility indexes

The VXO is calculated from the IV using the Black-Scholes model over a set of eight options on the S&P 100 index, four calls and four puts, close to ATM within the two nearest times to maturity. A detailed explanation about VXO calculation can be found in Whaley (1993). Briefly, the procedure consists of: (1) selecting the IV of each chosen option and multiplying it by a factor of $\sqrt{30/22}$ to compress the initial calendar volatility measure (on an actual/365 basis) in the approximate trading days. This first step has a drawback in that the volatility levels are artificially increased and the resulting index is upward biased. (2) The volatilities are averaged in successive steps until obtaining the VXO, first for each maturity (near and next) and strike (above and below the underlying price); calls and puts IV are averaged by averaging between the implied volatilities of calls and puts at each maturity. It is possible to some extent to reduce potential biases caused by the different reactions of both types of options to changes in the underlying. For example, when the market moves upward very quickly a positive (negative) bias is produced in the call (put). (3) The next step is to average the volatilities of the different strikes for each maturity. (4) Finally, the averaged values of each maturity are standardized to a 22-day period using linear interpolation:

where σ_1 is the averaged IV for the nearest maturity, T_1 is the number of days until the nearest maturity date, σ_2 is the averaged IV for the next time to maturity and T_2 is the number of days until the next time to maturity.

The VXO index is based on a weighted Black Scholes implied volatility model, and is, thus, highly dependent on this specific option valuation model. VXO index can be interpreted as an accurate approximation of the volatility swap rate, whose payoff is obtained with the realized volatility. However, it is hard to find a replica portfolio to hedge this volatility swap contract.

The VXO design was disputed by academics and practitioners, and on September 23, 2003, the CBOE adopted a new methodology for calculating the VIX index. This change involved two major developments: 1) replacing the S&P 100 for the S&P 500, based on the greater liquidity of the latter, 2) modifying the calculation method replacing the IVs with a weighted sum of OTM (out of the money) option prices.

The generalized formula used to calculate the daily variance is[1]:

$$\sigma^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} e^{rT} Q(K_i) - \frac{1}{T}\left(\frac{F}{K_o} - 1\right)^2$$

where:

- σ^2 the daily variance
- T the time to expiration
- F the forward index level derived from index option prices
- K_o the first strike below the forward index level, F
- K_i the strike price of i^{th} out of the money option; a call if $K_i > K_o$ and a put if $K_i < K_o$; both put and call if $K_i = K_o$
- ΔK_i interval between strike prices, half the difference between the strike on either side of K_i

$$\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}$$

[1] More information about VIX can be obtained at www.cboe.com

- r risk-free interest rate to expiration
- $Q(K_i)$ the midpoint of the bid-ask spread for each option with strike K_i

And finally, this is adjusted to obtain the VIX:

$$\text{VIX} = 100\sqrt{\frac{365}{12} Var_{30}}$$

With the new design, the VIX index is an approximation of the variance swap rate which payoff is obtained with the realized variance. This is one advantage of VIX because a variance swap contract can be hedged only with a static position in a portfolio of options and a dynamic position in futures trading.

Basically, the VIX approximates the variance swap rate on SPX and the old VXO approximates the volatility swap rate on OEX. Payoffs to variance swap contracts are much easier to replicate/hedge than payoffs to volatility swaps. Hence, CBOE switched to the new VIX and have indeed launched futures and options on the new VIX.

The procedure to calculate the German index, VDAX[2], is quite similar to the one for VXO but differs in two technical aspects. First, the reference period is 45 calendar days and no trading-day conversion is made. Second, the calculation for the ATM position is performed on the underlying forward price for the given maturity. Therefore, it is a requirement to calculate the forward price of the DAX (German stock index market). If a DAX future contract exists, whose maturity matches the given date, the price of it is taken as the forward price; otherwise it is calculated by linearly interpolating the prices of futures for the previous and following maturities.

For each maturity, similar to the VXO, four options are chosen (two calls and two puts), whose strikes are the closest above and below of the previously calculated forward price. These options are used to calculate the implied volatility, following the steps explained above for the VXO, except for the last one because a 45-day period is considered.

[2] More information about VDAX and VDAX-NEW can be obtained at www.dax-indices.com.

The VDAX-New (V1X code) is calculated identically to the VIX, but using the DAX German index, so no further explanation is required.

2.2 Historical analysis of the series

Using historical data of closing prices of options on the S&P 500 and the S&P 100 indexes, the CBOE has calculated the values of the VIX since 1990 and the VXO since

1987, and currently still continues to calculate both indexes. Deutsche Börse, for the German market, has calculated values of the VDAX and of the V1X since 1992 and it also continues to publish both indexes.

The common interval ranging from April 2, 1992 to December 30, 2011, which involves 4997 negotiation days, was chosen for this analysis. In addition to the four volatility indexes (VIX, VXO, VDAX and V1X), we analysed the 30 calendar days realized volatility for the market indexes: S&P 100 (VXO), S&P 500 (VIX) and DAX (V1X):

$$\sigma_{t+1,t+30}^{R} = \sqrt{\frac{1}{m-1}\sum_{k=1}^{m}\left(r_{t+k} - \overline{r}_{t,t+30}\right)^2} \quad (2.1)$$

where $r_{t+k} = \ln S_{t+k} / S_{t+k-1}$ is the daily return of the index at the moment $t+k$, representing S the daily close price on the date specified by the sub index, $\overline{r}_{t,t+30}$ is the 30 calendar day average return and m is the actual number of trading days for each period[3].

For the VDAX, a 45-calendar day realized volatility is considered:

$$\sigma_{t+1,t+45}^{R} = \sqrt{\frac{1}{m-1}\sum_{k=1}^{m}\left(r_{t+k} - \overline{r}_{t,t+45}\right)^2} \quad (2.2)$$

The above daily volatilities are annualized by multiplying by the root of 250.

[3] Traditionally a constant period of 22 trading days is used, but in this paper, the exact number of trading days for each of the 30 calendar-day period is used.

TABLE 1. MAIN STATISTICAL INDICATORS OF VOLATILITY INDEX AND REALIZED VOLATILITY FROM APRIL 2, 1992 TO DECEMBER 30, 2011

	USA					Germany			
	RV_{30D} S&P500	VIX	RV_{30D} S&P100	VXO	VXOA	RV_{30D} DAX	V1X	RV_{45D} DAX	VDAX
Mean	16.39	20.59	16.6	21.26	18.20	20.73	23.76	20.91	22.27
Stand. Dev.	10.17	8.59	10.1	9.38	8.03	10.99	10.22	10.65	9.04
Median	13.93	19.17	13.95	19.92	17.06	17.57	21.21	17.89	20.1
Excess Kurt.	10.92	6.26	9.61	5.53	5.53	3.65	2.84	3.22	2.27
Skewness	2.66	1.91	2.48	1.77	1.77	1.73	1.56	1.67	1.44
Minimum	4.73	9.31	4.5	9.04	7.74	4.68	9.35	5.83	9.36
Maximum	82.62	80.86	80.9	87.24	74.71	80.65	83.23	75.49	74

In Table 1, the main statistics of the four volatility indexes (VIX, VXO, V1X and VDAX) are summarised, as well as their corresponding 30-day realized volatility (45-days for VDAX). The mean and median values of the volatility indexes are higher than the mean and median values of the realized volatility of the indexes they are calculated on. Conversely, standard deviation, skewness and excess kurtosis are higher for realized volatility series than for the volatility indexes. Generally, the volatility levels show positive skewness and excess kurtosis, which can be explained by the arrival of significant leaps in the information series (Eraker et al., 2003).

Finally, it is also noteworthy that the minimum values of the four volatility indexes is around 9%, while the minimum volatility realized in the markets is around 5%. Thus, the differences between the indexes and the respective realized volatilities are nearly 100%. Maximum values of volatility reach about 80%.

Table 1 also includes statistics for VXOA, which takes into account the artificial upward bias used in VXO calculation, as noted by Carr and Wu (2006). So we have scaled back the conversion in the VXO, defining $\mathrm{VXOA} = \sqrt{22/30}\mathrm{VXO}$

Now, the results agree with the expected ones, VIX is higher than VXOA, as V1X is higher than VDAX. VIX calculation uses a square root of total variance $(\mathrm{E}[W_T])^{0.5}$, which is greater or equal to realized volatility $\mathrm{E}[(W_T)^{0.5}]$, as we can expect by the convexity adjustment argument.

The volatility indexes are considered a barometer of investor fear. The greater the fear, the greater the slope of market price movements is and the higher the index value. In this sense, taking as an example the VIX and the S&P 500 for the period 2000-2012 (see Figure 1), three clear periods can be highlighted:

1) Between the years 2000 and 2003, the dotcom bubble bursts, and the S&P 500 regresses from record highs of 1400 to 800 points, while the VIX increases from

20% to 45% in August 2002.

2) Between the years 2003 and 2007, the S&P 500 index slowly recovered the maximum values of the previous period to nearly 1500 points. In this stage of growth and stability (at least the absence of bad news), the VIX decreased to levels of 10%, close to the historical minimum registered on December 22th, 1993, 9.31%.

3) Between the years 2007 and 2010, as a result of the 2008 Financial Crisis, the S&P 500 index went down 45% from its 2007 high, and the VIX increased to a maximum of 80.86% on Nov 22th, 2008.

Indeed VIX can be used to predict future returns, with the predictive power being found to be more significant in a volatile bear market (Chung et al. 2011; Rosillo et al. 2014).

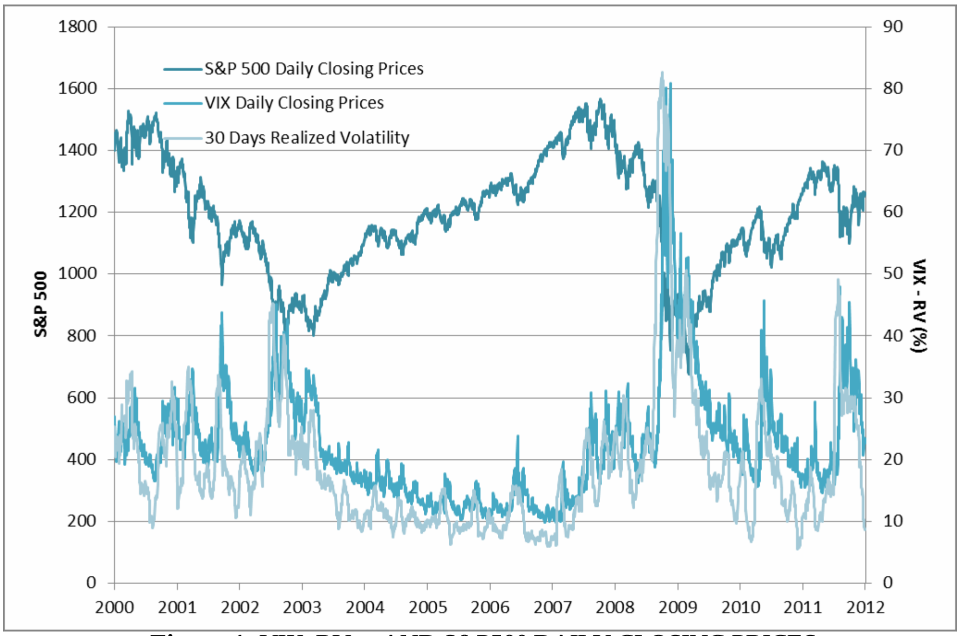

Figure 1. VIX, RV_{30D} AND S&P500 DAILY CLOSING PRICES JANUARY 1ST 2000 TO DECEMBER 31ST 2011

Similarly, figure 2 shows volatility index V1X, 30-day realized volatility and daily closing prices for the DAX German stock index.

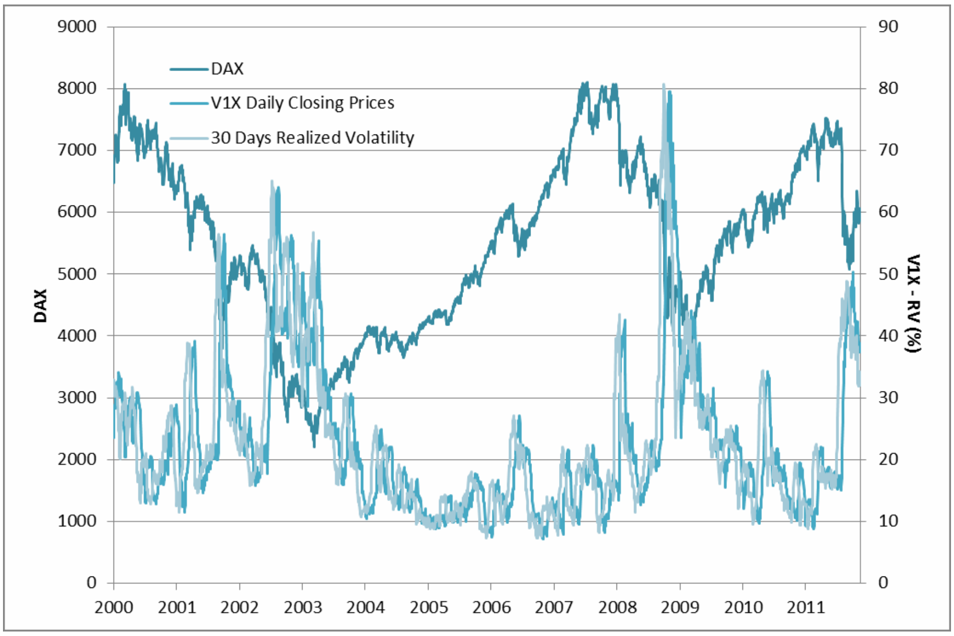

Figure 2. V1X, RV30D AND DAX DAILY CLOSING PRICES JANUARY 1ST 2000 TO DECEMBER 31ST 2011

In Figure 3, the differences between the VXOA and VIX indexes for the period April 2, 1992 to 30 December 2011 are shown. The average difference (VXOA - VIX) is, for the analysed period, -2.39, with a maximum of 3.69 in October 2008 and a minimum also in October 2008 of -11.17. In addition, the direction of the changes in both indexes matches 83% of the days.

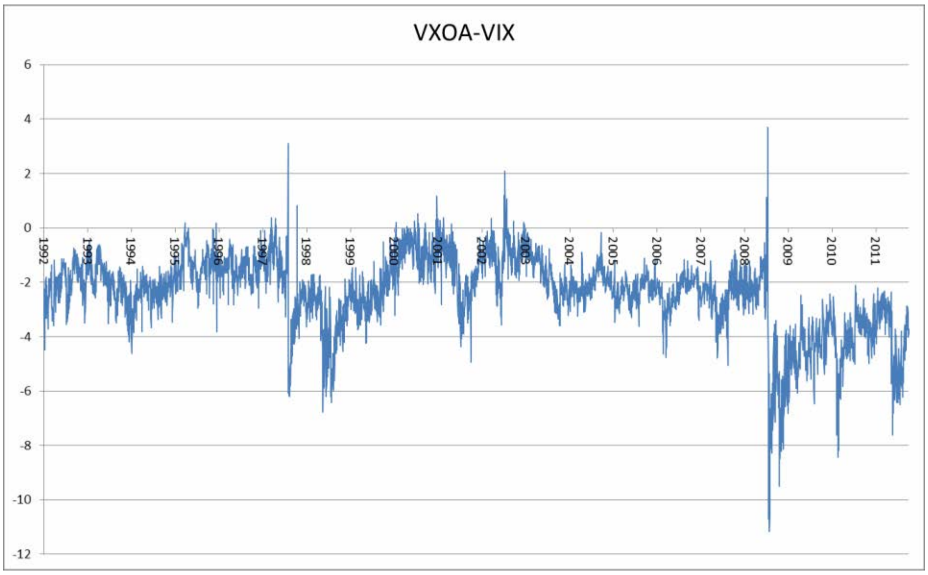

Figure 3. DIFFERENCES (VXO-VIX) IN THE PERIOD APRIL 2ND 1992 TO DECEMBER 30TH 2011.

The same analysis is performed using the VDAX and V1X, whose differences (VDAX - V1X) are represented graphically in Figure 4. The results show that the average difference is -1.48, with a maximum difference of 7.52 in October 1992 and a minimum of -20.43 in October 2008. The directional matching in the price movements is also 83%.

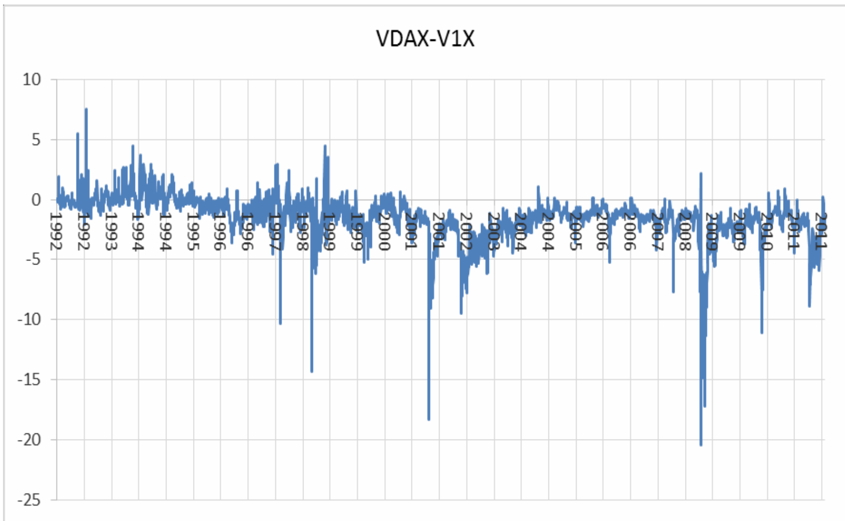

Figure 4. DIFFERENCES (VDAX-V1X) FROM APRIL 2ND 1992 TO DECEMBER 30TH 2011.

To evaluate whether the change in methodology provides truly different values, a mean difference test is performed. The null hypothesis is that there are no differences between the old and new values of the indexes. For the USA market, the null hypothesis is not accepted, so statistically significant differences exist between the values of the VXOA and VIX (p-value = 0.0000). For the German market, the same result is obtained (p-value = 1.495e-15). According to these results, the change in methodology has led to statistically significant different values, so in the next section we analyse whether these changes result in a greater predictive power.

3. VOLATILITY AND VARIANCE FORECASTING

In order to analyse whether the information embedded within the volatility indexes can be used to improve the volatility and variance predictions, we perform linear regressions and compare these results with two classical benchmarks: the historical volatility and the conditional volatility GARCH (1,1) model.

The informational content of the forecasts is gauged via Mincer-Zarnowitz type regressions of the observed T-day realized volatility on the corresponding forecast, $\sigma^{R}_{t+1,t+T} = \alpha + \beta\sigma^{j}_{t} + u_{t}$, where j denotes the j^{th} forecasting method.

Several authors, like Fleming et al. (1995), have observed that significant informational content exists in the implied volatility of the options. However, authors such as Canina and Figlewski (1993), Day and Lewis (1992) and Christensen and Prabhala (1998) claim not to have observed any correlation between the ATM implied volatility and realized volatility. These controversial results, as noted by Blair et al. (2001), can be explained since the implied volatility series used in these studies could contain significant measurement errors, whose magnitude is unknown.

Indeed, there may be many reasons that lead to this erroneous specification of implied volatility, but almost all are due to the complexity of the data: options are never completely at-the-money, there are very large bid-ask spreads in comparison to option prices, volatility is distorted by the impact of market commissions, the remaining time to maturity decreases with the life of the option, etc.. Another possible source of error is the use of an inappropriate option valuation model to evaluate the implied volatility, for example, using a European option model to assess American options, as is the case of the S&P100.

Regarding this, volatility indexes overcome some of the above problems, diminishing the measurement errors. All these considerations seem to point to a better performance of the VIX in forecasting the realised volatility or variance than other commonly used models, however, it must still be proved empirically.

The data used in this section are the daily returns of the market indexes: S&P 100, S&P 500 and DAX, along with the values for the analysed volatility indexes: VXO, VIX, V1X and VDAX. We analyse the common period 1992-2012, and we compare the realized volatility or variance of returns with the predictions from the different models used.

3.1 Prediction models

There is no general consensus on how to measure realized volatility. However, the usual way is to calculate realised volatility through the standard deviation of future performance, as shown by Andersen and Bollerslev (1998), Andersen et al. (2001) and Blair et al. (2001).

Our goal is to evaluate the usefulness of the volatility indexes studied in the previous section as forecasters of realized volatility or variance within the referred period. For the case of *T* calendar days, we would have:

$$\sigma^R_{t+1,t+T} = \sqrt{\frac{1}{m-1}\sum_{k=1}^{m}\left(r_{t+k} - \overline{r}_{t,t+T}\right)^2}$$

where r_{t+k} is the daily return of the market index (S&P 100, S&P 500 or DAX) at the moment $t+k$, and $\overline{r}_{t,t+T}$ is the average return of the *T* calendar days considered (with *T* equal to 30 or 45 calendar days, according to the analysed market index), and *m* is the effective number of trading days.

We are going to analyse the following regression models:

$$\sigma^R_{t+1,t+T} = \alpha + \beta\sigma^H_{t-(T-1),t} + u_t \qquad (3.1)$$

$$\sigma^R_{t+1,t+T} = \alpha + \beta\sigma^G_{t+1,t+T} + u_t \qquad (3.2)$$

$$\sigma^R_{t+1,t+T} = \alpha + \beta\,\mathrm{VX}_t + u_t \qquad (3.3)$$

where $\sigma^H_{t-(T-1),t}$ is the historical volatility of the T days prior to the close of the market on day t; $\sigma^G_{t+1,t+T}$, is the conditional volatility GARCH(1,1) predicted for T days later and VX_t is the close value of the volatility index on day t.

Since the VIX and V1X are designed to capture the variance expectations of their markets, we should expect VIX and V1X to forecast variance better and VXO and VDAX to forecast volatility better. In this sense, we also perform the analysis in terms of variance, in order to analyse in greater depth the relation between the realized variance and the proposed predictors, in this case: historical variance, conditional variance and squared volatility index.

$$(\sigma^R_{t+1,t+T})^2 = \alpha + \beta(\sigma^H_{t-(T-1),t})^2 + u_t \qquad (3.4)$$

$$(\sigma^R_{t+1,t+T})^2 = \alpha + \beta(\sigma^G_{t+1,t+T})^2 + u_t \qquad (3.5)$$

$$(\sigma^R_{t+1,t+T})^2 = \alpha + \beta\,\mathrm{VX}^2_{\mathrm{t}} + u_t \qquad (3.6)$$

Models (3.1) and (3.4) are based on historical volatility and act as a naïve benchmark. The historical volatility is calculated as the standard deviation of daily returns for the current day and the past T-1 days, $r_t, r_{t-1},...,r_{t-(T-1)}$:

$$\sigma^H_{t-(T-1),t} = \sqrt{\frac{1}{m-1}\sum_{k=1}^{m}\left(r_{t-k+1} - \overline{r}_{t-(T-1),t}\right)^2} \qquad (3.7)$$

The results are annualized by multiplying by $\sqrt{250}$.

Models (3.2) and (3.5) are based on GARCH (1,1) conditional volatility as proposed by Bollerslev (1986). We assume that the daily returns are distributed according to the model $r_t = C + \varepsilon_t$, where ε_t follows a $N\left(0,\sigma^2_t\right)$, C is a constant and σ_t is the conditional volatility characterized by

$$\sigma^2_t = \omega + \alpha_1\varepsilon^2_{t-1} + \beta_1\sigma^2_{t-1} \qquad (3.8)$$

Once the model is estimated, for each *t* we can make predictions of the conditional variance for the next *T* days, $\sigma^2_{t+1}, \sigma^2_{t+2}, \ldots, \sigma^2_{t+T}$, and we calculate the square root of their arithmetic mean:

$$\sigma^G_{t+1,t+T} = \sqrt{\frac{1}{T}\sum_{k=1}^{T} \sigma^2_{t+k}} \qquad (3.9)$$

This result is then annualized by multiplying by $\sqrt{250}$.

There are other conditional volatility models, such as the GJR (Glosten et al. 1993), which allows for asymmetries in the behaviour of volatility discriminating positive and negative shocks. It is also possible to include seasonal effects in the model that improve the characterization of the series. However, in this paper, we will not pursue this line of analysis so intensely.

Models (3.3) and (3.6) do not require further explanation. The volatility indexes are calculated by their respective markets.

Although, our main goal is the univariate analysis proposed through models (3.1) to (3.6), we also perform a multivariate analysis taking all the three forecast models: historical volatility, conditional volatility and volatility index (the same in variance):

$$\sigma^R_{t+1,t+T} = \alpha + \beta\,\mathrm{VX}_t + \gamma\sigma^G_{t+1,t+T} + \delta\sigma^H_{t-(T-1),t} + u_t \qquad (3.10)$$

$$\left(\sigma^R_{t+1,t+T}\right)^2 = \alpha + \beta\,\mathrm{VX}^2_t + \gamma\left(\sigma^G_{t+1,t+T}\right)^2 + \delta\left(\sigma^H_{t-(T-1),t}\right)^2 + u_t \qquad (3.11)$$

3.2 Empirical findings

The period analysed spans from April 1992 to December 2011, resulting in a sample with a total of 4997 trading days. As a previous step to performing models (3.2) and (3.5), the conditional variances must be estimated; the results for the GARCH (1,1) model in (3.8) are presented in Table 2. The analysis of the level of correlation of the standardized residuals using the Ljung-Box statistic and *t*-statistics are positive with regard to the modelling performed. Although the standardized residuals show significant autocorrelation (Ljung-Box Q(20), statistics exceed 31.5, at 5% critical value), the autocorrelations of squared standardized residual are no longer significant (Ljung-Box Q(20) statistics under 31.5).

Table 2. Estimated parameters and diagnostics of the GARCH (1,1) - model for S&P 100, S&P 500 and DAX in 1992-2012 period.

Index	***C***	$\boldsymbol{\omega}$	$\boldsymbol{\alpha 1}$	$\boldsymbol{\beta 1}$	***LBQTest***	***LBQTest2***
S&P 100	0.00053 (4.5)	9.8E-07 (8.2)	0.0795 (16.8)	0.9144 (179.7)	40.3	25.1
S&P 500	0.00053 (4.5)	8.9E-07 (7.5)	0.0780 (16.1)	0.9167 (174.7)	40.7	21.3
DAX	0.00071 (4.7)	2.3E-06 (7.8)	0.0897 (14.5)	0.9002 (137.8)	32.7	25.0

Note: LBQTest y LBQTest2: Ljung-Box Q(20) statístics for the standardized residuals and the squared standardized residuals (31.4 is the critical value at 5%)

All the previous models ((3.1) to (3.6)) are estimated by ordinary least squares (OLS). Table 3 depicts the estimated value for the parameters, their corresponding *t* statistics (calculated by the Newey and West (1987) method, since regression residuals are non-normal).

The statistical loss functions used are those implicit in various criteria and tests. First, the coefficient of determination R^2 is compared across forecasting methods. Second, defining the regression error as $e_{t+1,t+T} = \sigma^R_{t+1,t+T} - \hat{\sigma}^R_{t+1,t+T}$ for t=1, …,N we compute various statistical measures of accuracy: Mean Square Error, $MSE = \frac{1}{N}\sum_{t=1}^{N}(e_{t+1,t+T})^2$; Root Mean Square Error, $RMSE = \sqrt{MSE}$; Normalized Root Mean Square Error, $NRMSE = \frac{RMSE}{Desv(\sigma^R_{t+1,t+T})}$; Mean Absolute Error, $MAE = \frac{1}{N}\sum_{t=1}^{N}\left|e_{t+1,t+T}\right|$ and Heteroskedasticity-Adjusted MSE, $HMSE = \frac{1}{N}\sum_{i=1}^{N}(1-\hat{\sigma}^R_{t+1,t+T} / \sigma^R_{t+1,t+T})^2$.

Although the results of the regressions shown in Table 3 cannot be directly compared, since they refer to different markets (SPX or OEX in USA) or different periods (30 days or 45 days in Germany). We can, however, perform an intra-market analysis comparing forecasts.

Table 3. Prediction of 30 (45) day-ahead realised volatility (Panel A) and realised variance (Panel B) through regressions with historical volatility σ_H, conditional volatility GARCH(1,1) σ_G and volatility index.

Market	*Model*	α		β		γ		δ		R^2	*MSE*	*RMSE*	*NRMSE*	*MAE*	*HMSE*
Panel A: Volatility Models [(3.1) to (3.3) and (3.10)]															
S&P100	**σ_H**	4.4	(5.9)	0.74	(14.4)					0.540	46.8	6.84	0.68	4.52	0.15
	σ_G	1.0	(1.1)	0.90	(15.1)					0.584	42.4	6.51	0.64	4.27	0.13
	VXO	-1.4	-(1.6)	0.84	(18.0)					**0.615**	**39.2**	**6.26**	**0.62**	**3.92**	**0.10**
	VXO, σ_G, σ_H	-2.4	-(2.2)	0.57	(6.9)	0.76	(2.9)	-0.38	-(2.2)	0.632	37.5	6.12	0.61	3.85	0.12
S&P500	**σ_H**	4.1	(5.5)	0.75	(14.2)					0.556	46.0	6.78	0.67	4.41	0.14
	σ_G	1.0	(1.0)	0.90	(14.8)					0.598	41.6	6.45	0.63	4.16	0.13
	VIX	-2.7	-(2.8)	0.92	(17.8)					**0.610**	**40.4**	**6.35**	**0.62**	**3.90**	**0.11**
	VIX, σ_G, σ_H	-2.9	-(3.0)	0.53	(5.2)	0.85	(2.9)	-0.38	-(2.1)	0.634	37.9	6.16	0.61	3.80	0.12
DAX (45d)	**σ_H**	6.6	(9.8)	0.69	(19.7)					0.472	59.9	7.74	0.73	5.35	0.12
	σ_G	0.7	(0.8)	0.92	(22.3)					0.541	52.0	7.21	0.68	4.99	0.11
	VDAX	0.6	(0.9)	0.91	(28.0)					**0.598**	**45.6**	**6.75**	**0.63**	**4.48**	**0.08**
	VDAX, σ_G, σ_H	-1.9	-(2.0)	0.85	(12.6)	0.51	(4.6)	-0.36	-(3.8)	0.614	43.8	6.62	0.62	4.40	0.10
DAX (30d)	**σ_H**	5.7	(8.9)	0.73	(20.9)					0.529	56.8	7.54	0.69	5.24	0.13
	σ_G	0.2	(0.3)	0.94	(22.9)					0.582	50.5	7.10	0.65	4.93	0.12
	V1X	0.2	(0.4)	0.86	(29.4)					**0.645**	**42.9**	**6.55**	**0.60**	**4.44**	**0.09**
	V1X, σ_G, σ_H	-2.0	-(2.2)	0.78	(13.0)	0.53	(3.9)	-0.35	-(3.6)	0.653	41.9	6.47	0.59	4.39	0.09
Panel B: Variance Models [(3.4) to (3.6) and (3.11)]															
S&P100	**σ^2_H**	117.0	(4.4)	0.69	(7.4)					0.473	211964	460	0.73	213	1.94
	σ^2_G	52.0	(1.5)	0.86	(7.4)					**0.518**	**193907**	**440**	**0.69**	200	1.52
	VXO^2	-39.3	-(1.0)	0.77	(8.5)					0.506	198823	446	0.70	**189**	**0.81**
	VXO^2,σ^2_G,σ^2_H	-23.1	-(0.6)	0.34	(3.1)	0.90	(1.8)	-0.32	-(0.9)	0.541	184506	430	0.68	185	0.44
S&P500	**σ^2_H**	110.1	(4.0)	0.71	(7.2)					0.492	219903	469	0.71	207	1.86
	σ^2_G	49.9	(1.4)	0.86	(7.2)					**0.533**	**201843**	**449**	**0.68**	194	1.47
	VIX^2	-75.5	-(1.9)	0.90	(8.7)					0.504	214592	463	0.70	**190**	**0.92**
	VIX^2,σ^2_G,σ^2_H	-23.6	-(0.8)	0.31	(2.0)	0.97	(1.7)	-0.32	-(0.8)	0.548	195414	442	0.67	182	0.50
DAX (45d)	**σ^2_H**	220.0	(7.7)	0.61	(10.7)					0.366	275771	525	0.80	291	1.29
	σ^2_G	85.6	(3.2)	0.84	(14.7)					0.459	235385	485	0.74	269	1.07
	$VDAX^2$	41.6	(1.8)	0.88	(18.2)					**0.509**	**213271**	**462**	**0.70**	**239**	**0.66**
	$VDAX^2$,σ^2_G,σ^2	-2.0	-(0.1)	0.78	(9.2)	0.51	(3.7)	-0.34	-(2.9)	0.530	204511	452	0.69	236	0.62
DAX (30d)	**σ^2_H**	189.3	(8.4)	0.66	(13.2)					0.437	270225	520	0.75	283	1.56
	σ^2_G	66.3	(2.7)	0.88	(15.1)					0.498	240585	490	0.71	264	1.28
	$V1X^2$	30.4	(1.4)	0.78	(18.8)					**0.563**	**209747**	**458**	**0.66**	**234**	**0.77**
	$V1X^2$,σ^2_G,σ^2_H	-3.8	-(0.1)	0.69	(8.2)	0.42	(2.1)	-0.26	-(1.7)	0.569	206583	455	0.66	232	0.41

Note : *t* statistic of regression parameters calculated according to the Newey and West (1987) method. In bold the best univariate forecasting approach according to the loss function indicated in the column heading within each market.

In this sense, according to results in Table 3 – Panel A (volatility regressions), the accuracy of the forecasts for all the markets is best when volatility indexes are used to forecast and worst when the historical volatility is used. Thus model 3.3 outperforms the other two equivalent models, both in terms of coefficient of determination and the different statistical measures of accuracy computed. As expected, the results of multivariate model (3.10) are slightly better, but they do not lead to significant improvements, as other authors as Chung *et al.*, 2011 has noted before.

Results for models (3.4) to (3.6) and (3.11), see Table 3 – Panel B (variance regressions), indicate that in the USA market, conditional variance slightly outperforms the volatility index as a predictor for the most of the used loss functions. However, in the German market, clearly the prediction ability of the volatility index is higher. Again, the loss functions for multivariate model are slightly better.

We expected that volatility indexes model outperform the other forecasting models, in volatility and variance, but as we can see in Table 3 – Panel B (variance regressions), the VXO and the VIX indexes do not outperform the conditional variance regression model, showing a lower R^2 than the GARCH. This is a contradictory result, especially if we compare our results with those obtained by Carr and Wu (2006), who get a higher coefficient of determination for VIX regression (0.4687) than for GARCH regression (0.3576) in the period from January 2, 1990 to October 18, 2005.

We compare our results with those found by Carr and Wu (2006), and we perform the regression in two sub-periods: one from May 2, 1990 to October 18, 2005, which is approximately the period used by Carr and Wu (2006) and the other from October 19, 2005 to December 30, 2011. The results in Table 4 show that for the first sub-period the coefficient of determination is almost equal to that reported by Carr and Wu (2006). So, the difference detected could be due to the effect of the second sub-period in the whole sample. Indeed, for this sub-period, the GARCH estimation gives a higher coefficient of determination than VIX regression. In our opinion, this result is due to the 2008 Financial Crisis that has increased the volatility of the market, which is better captured by GARCH than VIX, since the former makes an in sample forecast and adapts the coefficients to this fact.

Table 4. Coefficient of determination for S&P500 variance regression in different sub-periods.

Variance Forecast	***R _Sub1***	***R^2_Sub2***	***R^2_All***
VIX^2	0.4581	0.4946	0.5038
σ^2_c	0.3647	0.5574	0.5333

SubPeriod1: May 2, 1990, to October 18, 2005
SubPeriod2: October 19, 2005, to December 30, 2011
All: April 2, 1992, to December 30, 2011

Finally, in order to assess the forecasting ability of each model, we have performed the Diebold-Mariano (DM) test. The null hypothesis is that the two models have the same forecast accuracy, and the alternative hypothesis is that model 1 and model 2 have different levels of accuracy. Should the latter be true, we test which of the models is more accurate. The results (in Table 5), for each index and pair of models indicates that the accuracy of the GARCH and the Volatility Index is greater than the accuracy of the Historical Volatility, in both volatility and in variance analysis. However, the relation between the accuracy of the GARCH model and Volatility Index model is not so straightforward. For the German Market, in volatility and variance, the Volatility Index model is more accurate than the GARCH model, but for the USA market except for VXO in volatility, the GARCH model and Volatility Index Model have the same level of accuracy.

Table 5. Results for Diebold-Mariano test

		Same accuracy		Greater accuracy		Same accuracy		Greater accuracy
Index	*Forecasts*	*DM*	*p-value*	*p-value*	*Forecasts*	*DM*	*p-value*	*p-value*
VXO	σ_H & σ_G	9.2326	<2.2E-16	<2.2E-16	σ^2_H & σ^2_G	4.7702	<2.2E-16	<2.2E-16
	σ_H & VX	9.2465	<2.2E-16	<2.2E-16	σ^2_H & VX^2	1.8665	0.06203	0.03102
	σ_G & VX	3.7019	0.0002163	0.0001082	σ^2_G & VX^2	-0.7035	0.4818	-
VIX	σ_H & σ_G	9.066	<2.2E-16	<2.2E-16	σ^2_H & σ^2_G	4.3916	1.15E-05	<2.2E-16
	σ_H & VX	6.548	6.41E-11	3.20E-11	σ^2_H & VX^2	0.7275	0.4669	-
	σ_G & VX	1.210	0.1969	-	σ^2_G & VX^2	-1.6224	0.1048	-
VDAX	σ_H & σ_G	10.486	<2.2E-16	<2.2E-16	σ^2_H & σ^2_G	8.0202	1.30E-15	<2.2E-16
	σ_H & VX	16.950	<2.2E-16	<2.2E-16	σ^2_H & VX^2	12.3426	<2.2E-16	<2.2E-16
	σ_G & VX	9.516	<2.2E-16	<2.2E-16	σ^2_G & VX^2	5.6592	1.61E-08	8.03E-09
VX1	σ_H & σ_G	10.933	<2.2E-16	<2.2E-16	σ^2_H & σ^2_G	7.2202	5.97E-13	<2.2E-16
	σ_H & VX	17.282	<2.2E-16	<2.2E-16	σ^2_H & VX^2	11.3090	<2.2E-16	<2.2E-16
	σ_G & VX	10.928	<2.2E-16	<2.2E-16	σ^2_G & VX^2	7.113	1.299E-12	6.493E-13

Same accuracy columns show the results for the two sided test. Greater accuracy columns show whether the second forecast has greater accuracy than first forecast (1st & 2nd)

Using the encompassing test of Harvey *et al.* (1998), we test if each of the univariate models encompasses the others. The results in Table 6 show, for each market and pair of model, the six possible combinations. Each column shows the *F* statistic and the *p*-value of the test that forecaster 1 encompasses forecaster 2 as null hypothesis. The results show that for the USA market, the null hypothesis is rejected, so no model encompasses any other; that is, including new information via historical volatility, GARCH volatility or index volatility improves the forecast of realized volatility (the same applies to variance). In the German market, the results are the same, except for V1X for which the Volatility index model encompasses the historical volatility model, so historical volatility does not add any information that improves the forecast of realized volatility (variance).

Table 6. Results for encompassing tests

Panel A: Volatility Models [(3.1) to (3.3)]							
		σ_H & σ_G	σ_G & σ_H	σ_H & VX	VX & σ_H	σ_G & VX	VX & σ_G
S&P 100	*F*	603.403	70.379	1030.917	50.182	565.47	150.52
	p-value	<2.2E-16	<2.2E-16	<2.2E-16	1.60E-12	<2.2E-16	<2.2E-16
S&P 500	*F*	602.958	71.682	816.500	110.560	409.500	250.930
	p-value	<2.2E-16	<2.2E-16	<2.2E-16	<2.2E-16	<2.2E-16	<2.2E-16
DAX (45d)	*F*	765.944	9.174	1596.571	19.866	760.820	46.272
	p-value	<2.2E-16	0.002467	<2.2E-16	8.49E-06	<2.2E-16	1.15E-11
DAX (30d)	*F*	671.666	37.889	1626.479	3.344	908.771	22.788
	p-value	<2.2E-16	8.076E-10	<2.2E-16	0.068	<2.2E-16	1.86E-06
Panel B: Variance Models [(3.4) to (3.6)]							
		σ^2_H & σ^2_G	σ^2_G & σ^2_H	σ^2_H & VX^2	VX^2 & σ^2_H	σ^2_G & VX^2	VX^2 & σ^2_G
S&P 100	*F*	537.506	68.419	523.6	183.08	205.33	336.51
	p-value	<2.2E-16	<2.2E-16	<2.2E-16	<2.2E-16	<2.2E-16	<2.2E-16
S&P 500	*F*	507.404	57.216	381.39	252.04	119.55	441.27
	p-value	<2.2E-16	4.62E-14	<2.2E-16	<2.2E-16	<2.2E-16	<2.2E-16
DAX (45d)	*F*	895.938	33.374	1493.479	23.169	576.251	52.928
	p-value	<2.2E-16	8.06E-09	<2.2E-16	1.53E-06	<2.2E-16	4.00E-13
DAX (30d)	*F*	659.8080	39.666	1443.4257	2.6834	751.6730	15.186
	p-value	<2.2E-16	3.27E-10	<2.2E-16	0.1015	<2.2E-16	9.87E-05

4. OPTIMAL PREDICTION PERIOD

As shown in the previous section, volatility indexes provide useful information about the expected volatility over the next standard period (30 or 45 days). However, to our knowledge, no research has been performed to test which is the optimal *T*-ahead. It may be the case that volatility indexes forecast the realized volatilities more accurately for a different period to 30 (or 45) days, for example a *T*, such as 10 days or maybe 40 days. To sum up, the question to answer is: what time horizon is the informational content of volatility indexes best adjusted for?

This is not a trivial issue, since the volatility index is the average of two volatilities that are the closest to the maturity and each reflect different expectations of investors. Investors are not necessarily considering a 30 day standard period, thus it is likely that the best forecasting does not match a 30-day period.

In order to test our hypothesis, we use models (3.3) and (3.6) from the above section, but the realized volatility is calculated following:

$$\sigma^R_{t+1,t+T} = \sqrt{\frac{1}{m-1}\sum_{k=1}^{m}\left(r_{t+k} - \overline{r}_{t,t+T}\right)^2}$$

considering T values from 5 to 90 days. In this way, for the entire period studied, we compute the realized volatility for each value of T, resulting in 86 series, one for each considered T. For each of these series, models (3.3) and (3.6) are tested for the four volatility indexes, which are not modified and have a constant maturity of 30 (or 45) days.

In Figure 5, we represent the coefficient of determination R^2 of the three regressions in the American market series, both for the VIX and the VXO, which show a similar behaviour with regard to the period they obtain the best forecast for. As can be seen in Figure 5, the best forecast is for 14 calendar days, although 30 calendar days is the period that the indexes are built for.

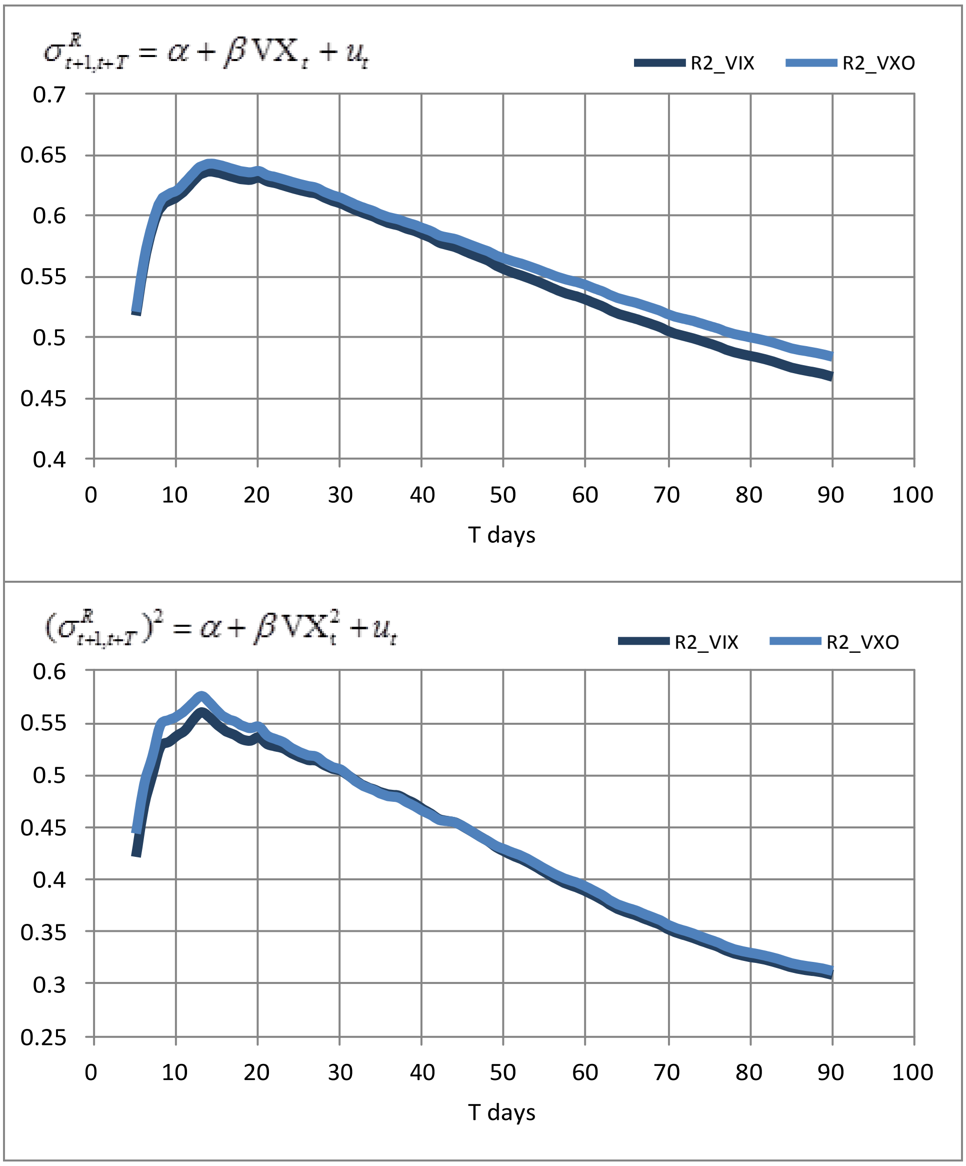

Figure 5. Coefficient of determination for different T day periods and regression models for VIX and VXO

Therefore, in view of the results (Figure 5), we can show that when a horizon of 14 days is reached, the additional information that is accumulated for computing the realised volatility no longer provides a substantial improvement in terms of prediction. Between 13 and 17 days, the difference in the forecast accuracy is less than 1%, thus, it can be said that both indexes (VXO and VIX) predict realized volatility better for this period, with the best fit being 14 days.

A similar behaviour is detected (Figure 6) for the German market, where the optimum forecasting period ranges from 18 to 27 calendar days for V1X and between 16 and 26 calendar days for the VDAX, in both cases the best forecast is recorded at 22 days.

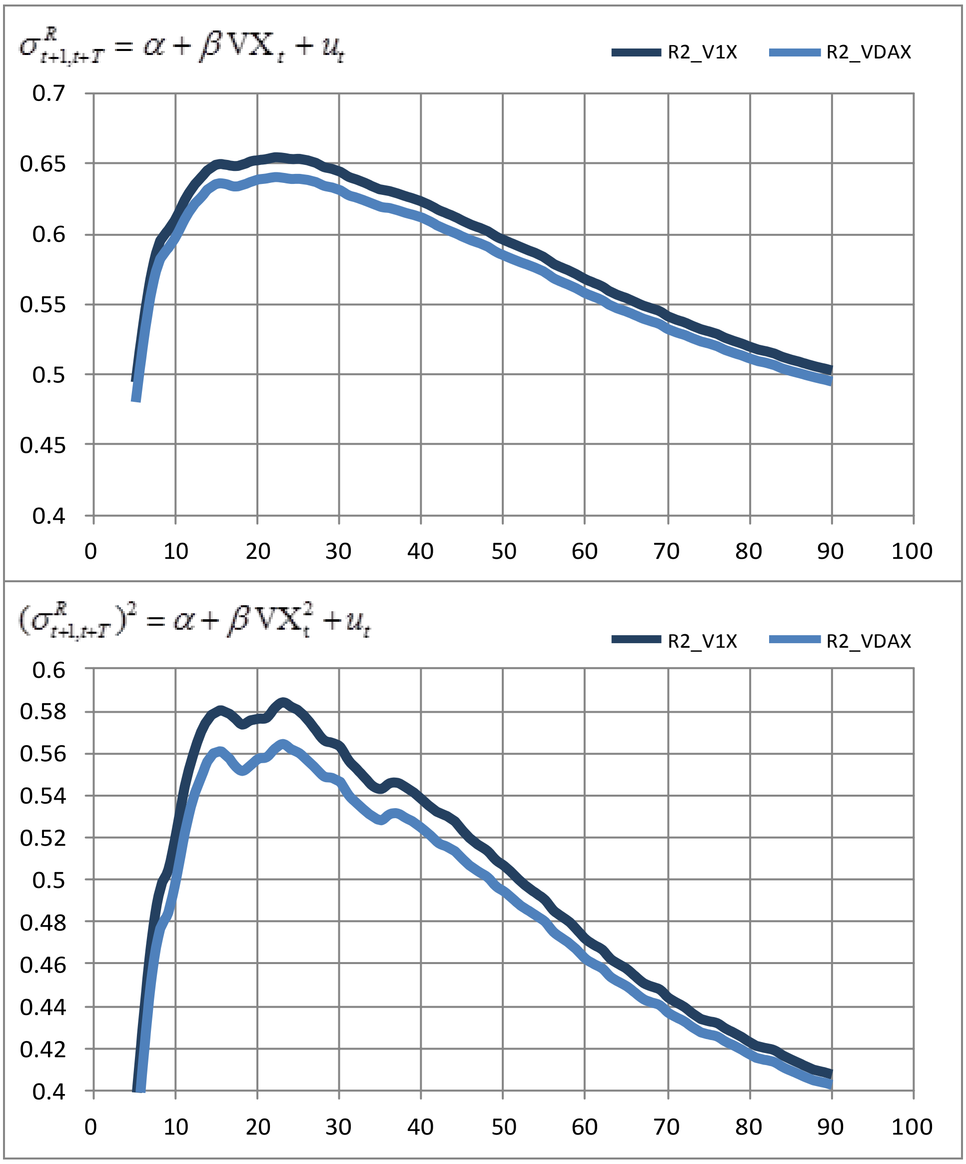

Figure 6. Coefficient of determination for different T day periods and regression models for V1X and VDAX

There are several reasons that could explain this unexpected result. One reason may be based on the nature of the indexes themselves, specifically in the particular way they are designed. Also the results could be related to investors' cognitive bias, which is largely analysed under the behavioural finance framework (Bird and Yeung, 2012). The underlying idea is that there is a time limit for which the investor is able to forecast the future price movements; that is, since volatility reacts quickly to any unexpected change in the market, it is not worth the investor trying to predict volatility for anything but very short periods.

A third explanation could be found in overreaction, previous empirical studies (Stein, 1989; Poteshman, 2001; Christoffersen et al., 2013) have shown that implied volatilities of long-term options react quite strongly to changes in implied volatilities of short-term options and do not display the rationally expected smoothing behaviour. Given the observed strong mean-reversion in volatility, these findings have been interpreted as evidence for overreaction in the options market. The evidence contradicts the rational expectation hypothesis for the term structure of implied volatility. It is observed that market participants do not take this fully into account when pricing options. Longer term options implied volatilities move almost in lockstep with those on shorter term options, displaying less of the "smoothing" behaviour than is warranted. In this sense, longer term options overreact relative to shorter term ones: they place too much emphasis on innovations in short-term options' implied volatility and too little emphasis on historical data that indicate that these innovations will not persist (Stein, 1989). Further, Poteshman (2001) concludes that stock market investors underreact to information at short horizons and overreact to information at long horizons.

According to Lehnert et al (2013), overreaction is explained by risk aversion level and volatility dynamics. Thus, when investors are highly risk averse, and risk-neutral volatility is highly persistent, the overreaction is higher. In contrast, in periods of low risk aversion, long-term volatility should react less strongly to changes in short-term volatility because risk-neutral volatility is less persistent. In this sense, very short maturities and very long maturities options are being mispriced and the volatility index, which is a weighted measure of variance, holds this bias. We assume that the measurement errors of both maturities compensate each other at the optimal periods.

5. CONCLUSIONS

In this paper, we analyse 20 years of daily data on four volatility indexes at two markets (USA and Germany) and we obtain several interesting findings on the indexes' behaviour. We find that the volatility indexes are better forecasters of realized volatility than the two, more frequently used, classical benchmarks (historical volatility and GARCH - conditional volatility), when the analysis is performed in terms of volatility. The results agree with those by other authors and confirm the better prediction accuracy of the volatility indexes. Moreover, the encompass test shows that for the American market, to include new information via historical volatility, GARCH volatility or index volatility improves the forecast of realized volatility (the same applies to variance), but for V1X the historical volatility (variance) does not add information that improves the forecast of realized volatility (variance).

Volatility indexes have been widely studied in the literature from different perspectives, but never before has the determination of the optimal forecast period of such indexes been attempted. Thus, we answer the question: Is the best forecast really 30 days ahead? We find there is no match between the index calculation period (usually 30 days) and the period of optimal prediction of realized volatility. In the American market, the results in terms of coefficient of determination suggest that investors do not attempt to predict future movements beyond two weeks. The explanation for this could be because volatility reacts quickly to any unexpected changes in the market; therefore, it is not worth the investor trying to predict volatility for anything but very short periods. In Germany, the optimum period is approximately three weeks. Both result are in line with overreaction analysis commented in the previous section but could also be due to the design of the indexes themselves.

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